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Multiple Diffraction by an Aperture in a Hard Screen

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Abstract

Two methods previously applied to diffraction by an aperture in a soft screen (one on which the field vanishes) are adapted to the case of a hard screen (one on which the normal derivative of the field vanishes). These methods are the geometrical theory of diffraction and the self consistent field method. They are both applied to diffraction by an infinite slit and both yield the same result at distances greater than several wave lengths from the edges of the screen. The former is also applied to diffraction by a circular aperture. Expressions for the diffracted fields, the diffraction patterns and the transmission cross sections are obtained for the problems of the slit and the circular aperture.

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Introduction

The present authors have previously presented two different methods for calculating the field diffracted when a wave hits an aperture in a thin screen [1], [2]. Both methods were applied to soft screens, i.e. screens on which the wave function vanishes. Some results were also given in [1] for hard screens on which the normal derivative of the wave function vanishes, but interaction between various parts of the edge of the screen was not taken into account. In the present article these two methods are extended to yield more complete results for hard screens, in that interaction is taken into account.

The geometrical theory of diffraction, described in reference [1], employs diffracted rays which are produced when a ray hits the edge of the screen. Some of the singly diffracted rays, which are produced by the incident rays, cross the aperture and hit the edge again, producing doubly diffracted rays, etc. The amplitude of the field on any diffracted ray is determined from that on the corresponding incident ray by means of a diffraction coefficient D which is given in [1]. In the case of a hard screen and an incident ray lying in the plane of the screen, D vanishes and the diffracted field is proportional to the normal derivative of the incident field at the edge. The proportionality factor is a new diffraction coefficient D' which has not been determined before. We will determine it and apply it to the diffracted fields produced by slits, circular apertures and other shaped apertures in hard screens.

The self-consistent field method described in [8], was applied in [2] to diffraction by an infinite slit. The field scattered by each edge of the slit was assumed to be produced by the incident field and by a cylindrical wave* emanating from the other edge. This assumption led to a pair of linear equations from which the amplitude of the field coming from each edge was determined. In the

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* i.e. the field of a line source.

case of a hard screen it does not suffice to treat the field scattered from one edge to the other as being equivalent to the field of a line source since then it would not interact with the second edge. Instead we consider this field to be that due to a line dipole. Then it does interact with the second edge and we can determine its strength as before. The resulting diffracted field coincides with that which we obtain by summing the fields on all the diffracted rays of the preceding method. This agreement lends support to both methods. The results also agree, where comparable, with those of H. Levine [3] and R.F. Millar [4] which were obtained by different methods.

The results of both methods given here do not apply to grazing incidence, although they can be modified to become applicable. Method II applies to the calculation of both far and near fields, provided that the interedge spacing is large. For an example of its application to the calculation of higher order interactions between arbitrary cylinders, in terms of their non-interaction patterns, see [7]. On the other hand, method I is applicable at any fixed point, in the limit of large frequency. In particular, both methods are applicable in the far field.

I. The geometrical theory of diffraction by a hard screen

1. The diffraction coefficient

In the geometrical theory of diffraction the field u_e on a ray diffracted from an edge is given by [1, eq.(12)]

$$u_e = D\left[s(1 + \rho_1^{-1}s)\right]^{-1/2} e^{iks} u_i \,. \tag{1}$$

Here u_i denotes the incident field at the point of diffraction on the edge, s denotes distance from this point along the diffracted ray, ρ_1 denotes distance to the caustic of the diffracted rays and D is the diffraction coefficient. For a scalar field u satisfying the reduced wave equation and incident upon a hard screen, on which $\partial u/\partial n = 0$, D is given by [1, eq.(13)]

$$D(\beta,\alpha) = -\frac{e^{i\pi/4}}{2(2\pi k)^{1/2}\sin\beta}\left[\sec\tfrac{1}{2}(\theta-\alpha) - \csc\tfrac{1}{2}(\theta+\alpha)\right] . \tag{2}$$

The angles α and θ are defined by projecting the incident and diffracted

rays onto the plane normal to the edge at the point of diffraction. The angles between these projections and the normal to the screen are α and θ respectively (see Fig. 1). The angle β is that between the incident ray and the edge. When the incident ray lies in the plane of the screen and $\alpha = \pi/2$, it follows from (2) that $D = 0$. In this case (1) must be replaced by [1, eq.(12')]

$$u_e = D'\left[s(1+\rho_1^{-1}s)\right]^{-1/2} e^{iks}\partial u_i/\partial n \quad . \tag{3}$$

Here $\partial u_i/\partial n$ denotes the normal derivative of the incident field at the point of diffraction and D' is another diffraction coefficient. We will now determine D' for scalar fields satisfying the reduced wave equation. To do so we seek a two dimensional field u satisfying $(\nabla^2+k^2)u=0$ and having a nonvanishing normal derivative on a half screen lying along the negative y axis. For the incident field u_o we take a nonuniform plane wave travelling parallel to the screen and towards its edge. The simplest such field having a non-zero normal derivative on the screen is

$$u_o = xe^{-iky} \quad .$$

We note that u_o can be expressed as the derivative of a plane wave with respect to its direction of propagation

$$u_o = -\frac{1}{ik}\left[\frac{\partial}{\partial\alpha} e^{ik(x\cos\alpha - y\sin\alpha)}\right]_{\alpha=\pi/2} \quad . \tag{4}$$

Let $v(\alpha)$ denote the solution of the half plane diffraction problem with the plane wave incident from the direction α. Then it follows from (4) that u is given by*

$$u = \frac{-1}{ik} v_\alpha\left(\frac{\pi}{2}\right) . \tag{5}$$

We now apply (5) to the diffracted part of the field. Since $\partial u_o/\partial x = 1$, it follows that

$$D'(\theta) = -\frac{1}{ik} D_\alpha\left(\theta,\frac{\pi}{2}\right) \quad . \tag{6}$$

If the incident ray makes the angle β with the edge then the factor k in (6) must be replaced by $k\sin\beta$. Then from (6) and (2) we obtain

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* For the application of this device in more general multiple scattering problems see [7].

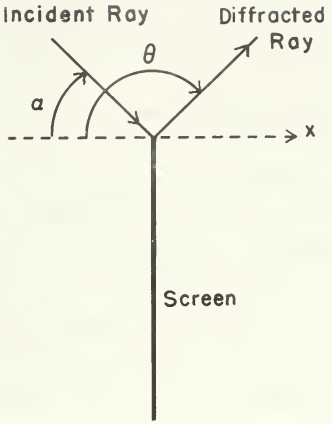

Fig. 1. The projection of incident and diffracted rays into a plane normal to the edge. The angles α and θ are the angles between these projections and the negative x axis, measured as shown in the figure. The diffracting edge, which is perpendicular to the plane of the figure, is the edge of the screen.

Table I.

Type	Initial diffraction	Number of diffractions at each edge: Lower	Number of diffractions at each edge: Upper	Final diffraction
A	Lower	n+1	n	Lower
B	Lower	n	n	Upper
C	Upper	n	n+1	Upper
D	Upper	n	n	Lower

$$(7) \qquad D'(\theta) = -\frac{e^{-i\pi/4}}{2(2\pi)^{1/2}k^{3/2}\sin^2\beta}\ \frac{\sin(\frac{\pi}{4}-\frac{\theta}{2})}{\cos^2(\frac{\pi}{4}-\frac{\theta}{2})} \ .$$

Upon combining (3) and (7), we finally obtain for the field on an edge diffracted ray

$$(8) \qquad u_e = \frac{e^{iks-i\pi/4}}{2(2\pi k^3)^{1/2}\sin^2\beta}\left[s(1-\frac{s|\underline{\cos}\delta+\rho\dot{\beta}\underline{\sin}\beta|}{\rho\sin^2\beta})\right]^{-1/2}\frac{\sin(\frac{\pi}{4}-\frac{\theta}{2})}{\cos^2(\frac{\pi}{4}-\frac{\theta}{2})}\ \frac{\partial u_i}{\partial n} \ .$$

In (8) we have introduced the expression for ρ_1 given in equation (11) of reference [1]. The angle δ is that between the diffracted ray and the normal to the edge, $\dot{\beta}$ is the derivative of β with respect to arclength along the edge and ρ is the radius of curvature of the edge.

If the edge has a corner then the field u_c on a corner diffracted is given by

$$(9) \qquad u_c = C'\frac{e^{iks}}{s}\ \frac{\partial u_i}{\partial n} \ .$$

This replaces equation (15) of reference [1] and C' is a new corner diffraction coefficient. From dimensional considerations we see that C' must be proportional to k^{-2}. We shall not consider this coefficient further.

The total field at a point is, as usual, the sum of the fields on all rays through the point. Let us now apply these results to some problems of diffraction through apertures.

2. Diffraction by a slit

Let us consider a plane wave incident upon a hard screen containing a slit of width 2a and of infinite length. The incident rays are assumed to be normal to the edges of the slit so the problem is essentially two dimensional. The singly diffracted field has been computed in reference [1] by using (1) on each of the two singly diffracted rays through any point. In attempting to determine the doubly diffracted field, we find that the singly diffracted rays which produce it lie in the plane of the screen. Therefore (1) is no longer applicable and (3) or (8) must be used.

To apply (8) we note that in the present case $\beta = \pi/2$, $\beta' = 0$, $\rho = \infty$ and $\rho_1 = \infty$. If the incident field is $\exp[ik(x\cos\alpha - y\sin\alpha)]$ and if the screen lies in the plane $x = 0$, application of (1) yields for the singly diffracted field

$$(10)\quad u_{el} = D(\theta_1,\alpha)r_1^{-1/2}e^{ik(r_1-a\sin\alpha)} + D(\theta_2,\alpha)r_2^{-1/2}e^{ik(r_2+a\sin\alpha)} .$$

In (10), r_1 and r_2 denote the distances of a point from the upper and lower edges $(0,\pm a)$ respectively. The angles θ_1 and θ_2 are determined by the corresponding rays as shown in Fig. 2. The field (10) was derived and examined in reference [1].

To determine the doubly diffracted field, let us consider that ray which is singly diffracted at the lower edge and which traverses the slit to the upper edge. The normal derivative of the field on this ray when it reaches the upper edge can be obtained from the second term in (10) by applying $(2a)^{-1}\partial/\partial\theta_2$ and then setting $r_2 = 2a$ and $\theta_2 = \pi/2$. Doing this yields

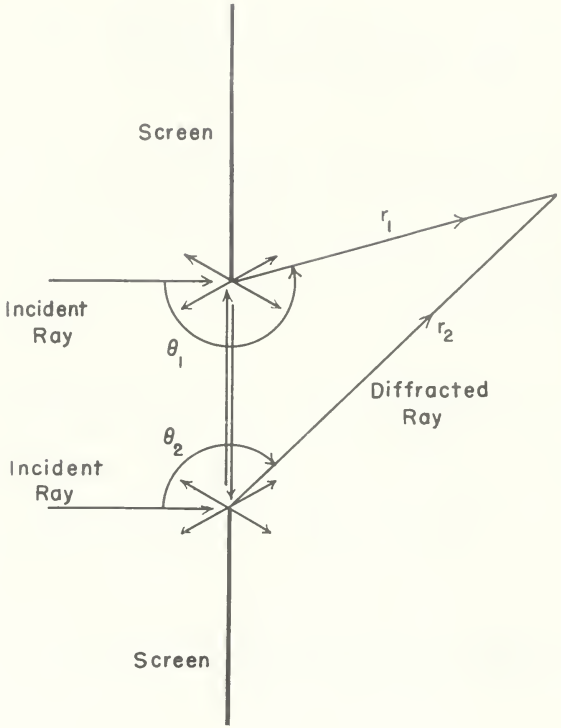

Fig. 2. Diffracted rays produced when a normally incident plane wave hits a slit in a thin screen. Some singly diffracted rays are shown leaving each edge. Two of these rays are shown crossing the slit and hitting the opposite edge where they produce doubly diffracted rays.

(11) $$\frac{\partial v_{el}}{\partial n} = -\frac{e^{ika(2+\sin\alpha)+i\pi/4}}{8(\pi ka^3)^{1/2}} \quad \frac{\sin(\frac{\pi}{4}-\frac{\alpha}{2})}{\cos^2(\frac{\pi}{4}-\frac{\alpha}{2})} \quad .$$

A similar expression with $-\alpha$ in place of α gives the normal derivative at the lower edge of the field on the ray singly diffracted at the upper edge. Insertion of either of these expressions into (8) yields the field on the doubly diffracted rays. Upon combining these two fields, we have for the doubly diffracted field

(12) $$u_{e2} = -\frac{e^{ika(2+\sin\alpha)+ikr_1}}{16\pi(ka)^{3/2}(2kr_1)^{1/2}} \quad \frac{\sin(\frac{\pi}{4}-\frac{\alpha}{2})\sin(\frac{\pi}{4}-\frac{\theta_1}{2})}{\cos^2(\frac{\pi}{4}-\frac{\alpha}{2})\cos^2(\frac{\pi}{4}-\frac{\theta_1}{2})}$$

$$-\frac{e^{ika(2-\sin\alpha)+ikr_2}}{16\pi(ka)^{3/2}(2kr_2)^{1/2}} \quad \frac{\sin(\frac{\pi}{4}+\frac{\alpha}{2})}{\cos^2(\frac{\pi}{4}+\frac{\alpha}{2})} \quad \frac{\sin(\frac{\pi}{4}-\frac{\theta_2}{2})}{\cos^2(\frac{\pi}{4}-\frac{\theta_2}{2})} \quad .$$

Far from the slit (12) simplifies in terms of polar coordinates r and ϕ. Then $r_1 \sim r-a\sin\phi$, $r_2 \sim r+a\sin\phi$, $\theta_1 \sim \pi+\phi$ and $\theta_2 \sim \pi-\phi$. Now (12) can be written as

(13) $$u_{e2} = -(k/2\pi r)^{1/2} e^{ikr+i\pi/4} f_d(\phi) \quad .$$

In (13) $f_d(\phi)$ denotes the contribution of double diffraction to the radiation or diffraction pattern of the slit. From (12)

$$(14) \qquad f_d(\phi) = -\frac{e^{2ika-i\pi/4}}{16\pi^{1/2}(ka)^{3/2}k}\left[\frac{\sin(\frac{\pi}{4}-\frac{\alpha}{2})\sin(\frac{\pi}{4}+\frac{\phi}{2})}{\cos^2(\frac{\pi}{4}-\frac{\alpha}{2})\cos^2(\frac{\pi}{4}+\frac{\phi}{2})}\,e^{ika(\sin\alpha-\sin\phi)}\right.$$

$$\left. + \frac{\sin(\frac{\pi}{4}+\frac{\alpha}{2})\sin(\frac{\pi}{4}-\frac{\phi}{2})}{\cos^2(\frac{\pi}{4}+\frac{\alpha}{2})\cos^2(\frac{\pi}{4}-\frac{\phi}{2})}\,e^{ika(-\sin\alpha+\sin\phi)}\right] .$$

The diffraction pattern due to single and double diffraction is $f_s(\phi) + f_d(\phi)$, where $f_s(\phi)$, which can be obtained from (10), is given by equation (23) of reference [1].

The contribution of double diffraction to the transmission cross section σ of the slit (per unit length) is equal to the imaginary part of $f_d(-\alpha)$, according to the cross section theorem. This can be computed from (14) and added to 2a cos α, which is the contribution of the transmitted and singly diffracted rays. There results

$$(15) \qquad \sigma = 2a\cos\alpha - \frac{2a}{32\pi^{1/2}(ka)^{5/2}}\left[\frac{\sin^2(\frac{\pi}{4}-\frac{\alpha}{2})}{\cos^4(\frac{\pi}{4}-\frac{\alpha}{2})}\sin\left[2ka(1+\sin\alpha)-\frac{\pi}{4}\right]\right.$$

$$\left. + \frac{\sin^2(\frac{\pi}{4}+\frac{\alpha}{2})}{\cos^4(\frac{\pi}{4}+\frac{\alpha}{2})}\sin\left[2ka(1-\sin\alpha)-\frac{\pi}{4}\right]\right] .$$

In the case of normal incidence $\alpha=0$ and (15) becomes

$$(16) \qquad \frac{\sigma}{2a} = 1 - \frac{\sin(2ka-\frac{\pi}{4})}{8\pi^{1/2}(ka)^{5/2}} .$$

To determine the field on any multiply diffracted ray we make repeated use of (8). In Table I on page 3a we have enumerated the four types of multiply diffracted rays. By referring to this table and using (8) we easily find that the field on a multiply diffracted ray of any type is given by one of the following expressions

(A) $$D'(\theta_2)r_2^{-1/2}e^{ikr_2}\left\{\left[D'_\theta(\pi/2)\right]^2(2a)^{-3}e^{4ika}\right\}^{n-1}D'_\theta(\pi/2)D_\theta(\pi/2,\alpha)(2a)^{-3}e^{ika(4+\sin\alpha)}.$$

(B) $$D'(\theta_1)r_1^{-1/2}e^{ikr_1}\left\{\left[D'_\theta(\pi/2)\right]^2(2a)^{-3}e^{4ika}\right\}^{n-1}D_\theta(\pi/2,\alpha)(2a)^{-3/2}e^{ika(2+\sin\alpha)}.$$

(C) $$D'(\theta_1)r_1^{-1/2}e^{ikr_1}\left\{\left[D'_\theta(\pi/2)\right]^2(2a)^{-3}e^{4ika}\right\}^{n-1}D'_\theta(\pi/2)D_\theta(\pi/2,-\alpha)(2a)^{-3}e^{ika(4-\sin\alpha)}$$

(D) $$D'(\theta_2)r_2^{-1/2}e^{ikr_2}\left\{\left[D'_\theta(\pi/2)\right]^2(2a)^{-3}e^{4ika}\right\}^{n-1}D_\theta(\pi/2,-\alpha)(2a)^{-3/2}e^{ika(2-\sin\alpha)}.$$

The total multiply diffracted field u_m is obtained by adding together the fields on the four types of rays and summing with respect to n. The series to be summed is a geometric series given by

(17) $$\sum_{n=1}^{\infty}\left\{\left[D'_\theta(\pi/2)\right]^2(2a)^{-3}e^{4ika}\right\}^{n-1}=\left[1-\left[D'_\theta(\pi/2)\right]^2(2a)^{-3}e^{4ika}\right]^{-1}.$$

Thus we obtain

$$(18)\quad u_m = \left[1-\left[D'_\Theta(\pi/2)\right]^2(2a)^{-3}e^{\frac{1}{4}ika}\right]^{-1}\left[D'(\theta_1)r_1^{-1/2}e^{ikr_1}\left\{D_\Theta(\pi/2,\alpha)(2a)^{-3/2}e^{ika(2+\sin\alpha)}\right.\right.$$

$$\left. + D'_\Theta(\pi/2)D_\Theta(\pi/2,-\alpha)(2a)^{-3}e^{ika(\frac{1}{4}-\sin\alpha)}\right\}$$

$$+ D'(\theta_2)r_2^{-1/2}e^{ikr_2}\left\{D_\Theta(\pi/2,-\alpha)(2a)^{-3/2}e^{ika(2-\sin\alpha)}\right.$$

$$\left.\left. + D'_\Theta(\pi/2)D_\Theta(\pi/2,\alpha)(2a)^{-3}e^{ika(\frac{1}{4}+\sin\alpha)}\right\}\right] .$$

The complete diffracted field $u_d = u_{el} + u_m$ is given by (10) and (18). We have also obtained exactly this same result by the method of Part II.

Far from the slit (18) becomes

$$(19)\qquad u = -(k/2\pi r)^{1/2}e^{ikr+i\pi/4}f_m(\phi) .$$

In (19) we have introduced $f_m(\phi)$, the diffraction pattern due to multiple diffraction. From (18) it is found to be given by

$$(20)\qquad f_m(\phi) = F(\phi,\alpha) + F(-\phi,-\alpha).$$

Here $F(\phi,\alpha)$ is defined by

$$(21)\quad F(\phi,\alpha) = -a\left[1+\frac{ie^{4ika}}{256\pi(ka)^3}\right]^{-1}\frac{\sin(\frac{\pi}{4}+\frac{\phi}{2})}{\cos^2(\frac{\pi}{4}+\frac{\phi}{2})}\left\{\frac{e^{ika(2+\sin\alpha-\sin\phi)-i\pi/4}}{16\pi^{1/2}(ka)^{5/2}}\cdot\frac{\sin(\frac{\pi}{4}-\frac{\alpha}{2})}{\cos^2(\frac{\pi}{4}-\frac{\alpha}{2})}\right.$$

$$\left.+\frac{ie^{ika(4-\sin\alpha-\sin\phi)}}{256\pi(ka)^4}\cdot\frac{\sin(\frac{\pi}{4}+\frac{\alpha}{2})}{\cos^2(\frac{\pi}{4}+\frac{\alpha}{2})}\right\}.$$

When $\alpha = 0$, (20) simplifies to

$$(22)\quad f_m(\phi) = -2a\left[1+\frac{ie^{4ika}}{256\pi(ka)^3}\right]^{-1}\left[\frac{e^{2ika-i\pi/4}}{8\pi^{1/2}(ka)^{5/2}}+\frac{ie^{4ika}}{128\pi(ka)^4}\right]$$

$$\propto \frac{1}{\cos^2\phi}\left[(\cos\tfrac{\phi}{2}+\sin\phi\sin\tfrac{\phi}{2})\cos(ka\sin\phi) - i(\sin\tfrac{\phi}{2}+\sin\phi\cos\tfrac{\phi}{2})\sin(ka\sin\phi)\right].$$

The transmission cross section of the slit, per unit length, is the imaginary part of $F_s(-\alpha) + F_m(-\alpha)$. For normal incidence ($\alpha = 0$) this yields

$$(23)\quad \frac{\sigma}{2a} = 1 - \left[\frac{\sin(2ka-\pi/4)}{8\pi^{1/2}(ka)^{5/2}}\left(1+\frac{1}{256\pi(ka)^3}\right)+\frac{\cos(4ka)}{128\pi(ka)^4}\right]$$

$$\propto \left[1-\frac{\sin(4ka)}{128\pi(ka)^3}+\frac{1}{(256\pi(ka)^3)^2}\right]^{-1}.$$

It should be noted that in the foregoing analysis, just as in that of reference [1], only the leading term in the expression for the field on a ray has been employed. Actually the field on any ray is given by a series

in powers of k^{-1}. The coefficients of the various powers can be determined from the first term by solving the transport equations, supplemented by appropriate initial conditions. A method for obtaining these initial conditions and thence the higher terms, is presented in reference [5] . For the slit we can obtain the correction terms on the singly diffracted rays directly from the Sommerfeld solution for diffraction by a half-plane. The field (10), augmented by the first correction term on each ray is

$$(24) \qquad u_{el} = -\frac{e^{ik(r_1 - a\sin\alpha)+i\pi/4}}{2\sqrt{2\pi kr_1}} \left[\sec\tfrac{1}{2}(\theta_1+\alpha) \pm \csc\tfrac{1}{2}(\theta_1-\alpha) + \frac{e^{-i\pi/2}}{4kr_1} \left\{ \sec^3\tfrac{1}{2}(\theta_1+\alpha) \pm \csc^3\tfrac{1}{2}(\theta_1-\alpha) \right\} \right]$$

$$- \frac{e^{ik(r_2+a\sin\alpha)+i\pi/4}}{2\sqrt{2\pi kr_2}} \left[\sec\tfrac{1}{2}(\theta_2-\alpha) \pm \csc\tfrac{1}{2}(\theta_2+\alpha) + \frac{e^{-i\pi/2}}{4kr_2} \left\{ \sec^3\tfrac{1}{2}(\theta_2-\alpha) \pm \csc^3\tfrac{1}{2}(\theta_2+\alpha) \right\} \right] .$$

The lower sign in (24) applies to a hard screen, the upper sign, which applies to a soft screen, is included for completeness. Far from the screen (24) becomes

$$(25)\quad u_{el} = \frac{-e^{ikr+i\pi/4}}{\sqrt{2\pi kr}} \left\{ i\frac{\sin\left[ka(\sin\phi+\sin\alpha)\right]}{\sin\frac{1}{2}(\phi+\alpha)}\left[1+\frac{1}{4ikr\sin^2\frac{1}{2}(\phi+\alpha)}\right] \right.$$

$$\left. \pm \frac{\cos\left[ka(\sin\phi+\sin\alpha)\right]}{\cos\frac{1}{2}(\phi-\alpha)}\left[1+\frac{1}{4ikr\cos^2\frac{1}{2}(\phi-\alpha)}\right] \right\} .$$

3. Diffraction by a circular aperture

Let us now consider a plane wave normally incident upon a circular aperture of radius a in a hard screen. The singly diffracted field consists of two terms, as in (10), and is given in reference [1], eq. (36). To determine the doubly diffracted field we must calculate the normal derivative of the singly diffracted field on a ray which has crossed the aperture to the opposite edge. From [1, eq. (36)] we obtain for this derivative

$$(26)\quad \frac{\partial u_{el}}{\partial n} = -\frac{e^{2ika-i\pi/4}}{4(2\pi ka^3)^{1/2}} .$$

Now upon using (26) in (8), and taking account of the two doubly diffracted rays through any point not on the axis, we obtain for the doubly diffracted field

$$(27)\quad u_{e2} = \frac{-ie^{ik(r_1+2a)}}{16\pi(k^4a^3)^{1/2}}\left[r_1(1-a^{-1}r_1\cos\delta_1)\right]^{-1/2}\frac{\sin\delta_1/2}{\cos^2\delta_1/2}$$

$$-\frac{ie^{ik(r_2+2a)}}{16\pi(k^4a^3)^{1/2}}\left[r_2(1-a^{-1}r_2\cos\delta_2)\right]^{-1/2}\frac{\sin(\delta_2/2)}{\cos^2(\delta_2/2)} .$$

At points near the axis where $\delta_1 \simeq \delta_2 \simeq \delta = \tan^{-1}(x/a)$, we let ρ denote distance from the axis and x denote distance from the screen. Then (27) simplifies to

$$(28)\quad u_{e2} = -\frac{a^{1/2} e^{ik(x\sin\delta + a\cos\delta + 2a) + i\pi/4}}{8\pi(k^4 a^3 \rho)^{1/2}(x^2+a^2)^{1/4}} \quad \frac{\sin(\delta/2)}{\cos^2(\delta/2)} \quad \cos(k\rho\cos\delta - \pi/4) \ .$$

The field (28) becomes infinite on the axis, which is a caustic of the diffracted rays. Upon applying the axial caustic correction to (28) it becomes

$$(29)\quad u_{e2} = -\frac{(a\cos\delta)^{1/2}\sin(\delta/2)}{8(2\pi k^3 a^3)^{1/2}(x^2+a^2)^{1/4}\cos^2(\delta/2)} \quad e^{ik(x\sin\delta + a\cos\delta + 2a) + i\pi/4} \ J_0(k\rho\cos\delta) \ .$$

Far from the screen δ tends to $\pi/2$ and this becomes

$$(30)\qquad u_{e2} = -\frac{aJ_0(ka\sin\phi)}{8x(\pi k^3 a^3)^{1/2}} \quad e^{ik(x+2a)+i\pi/4} \ .$$

To obtain the transmission cross section of the aperture we may apply the cross section theorem to $u_g + u_{e1} + u_{e2}$, where u_g denotes the geometrical optics field on the directly transmitted rays. From u_g and the leading term in u_{e1} we obtain the geometrical cross section πa^2. From the second term in u_{e1} given in [5] we obtain the correction $-\pi/4k^2$. When we add these contributions to that of u_{e2}, which we find from (30), we obtain

$$(31)\qquad \frac{\sigma}{\pi a^2} = 1 - \frac{1}{4(ka)^2} + \frac{\sin(2ka+\pi/4)}{4\pi^{1/2}(ka)^{5/2}} \ .$$

This result was previously obtained by H. Levine and T.T. Wu [6]. Additional terms can be obtained by considering further diffracted rays.

II. The self-consistent field method for a hard screen.

1. The method

The self-consistent field method* of solving diffraction problems is a method which is applicable to diffraction by a collection of simple objects or by a composite object. In it the field is assumed to consist of the incident field and a number of scattered or diffracted fields, one emanating from each object. The self-consistency condition is that each scattered field must be that which the object from which it emanates would scatter in the absence of the other objects, when all the other fields are incident upon it. If this condition were fulfilled exactly, the assumed field would be the exact solution of the problem. The method becomes approximate, and at the same time practical, when the field scattered from each object can be assumed to have (in the neighborhood of each other object) a specific form with an undetermined amplitude. For example, take the case when the spacing between objects is large compared to both the wave length and the transverse body dimensions. Then, such a form is available. The field arriving from each other object is then, in fact, necessarily expressible as the field of a linear combination of a finite number of sources and multipoles. If in addition, the spacing is large compared to all other body dimensions, the above form reduces to a combination of plane waves, and derivatives of plane waves with respect to incidence angle. In either event, the consistency condition leads to a set of linear algebraic equations for the amplitudes of the scattered fields. The coefficient matrix of this system of algebraic equations is expressible in terms of the response of each isolated body to a source or to a plane wave, and also in terms of the derivatives of those responses with respect to source postion or angle of incidence respectively. For another example, note that a similar simplification ensues when the condition of large spacing is replaced by the condition that all the objects are small compared to wavelength, since such bodies act like isotropic line sources or dipoles as far as their radiation is concerned.

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* c.f. [8].

In the case of a slit of infinite length in a thin hard screen, the simple objects referred to above are the two half-planes into which the slit divides the screen. The field is assumed to consist of an incident plane wave plus the two fields which the half planes scatter when they are hit by a plane wave and an additional scattered wave emanating from each half-plane. Each of these latter waves, which arise because of interaction between the two half planes, is assumed to be a wave of the form which the half plane would scatter if hit by the field from a line dipole located at the edge of the other screen. The amplitudes of these two waves are then determined by the consistency condition.

In a previous application of this method [2] to diffraction by a slit in a soft screen, a simpler form of scattered wave was employed. This was the field scattered by a half plane when hit by the field from an isotropic line source located at the edge of the other screen. Such an incident field is not scattered at all by a hard screen since it has a vanishing normal derivative on the screen. Therefore we must take into account a higher order emanation from each screen, namely a line dipole field. Such a field does not have a vanishing normal derivative on the screen. Then a non-zero scattered field results.

Once the amplitudes are determined, the field is known at all points of space, provided the spacing is large. From it the diffraction pattern and the transmission cross section per unit length can be found.

2. Discussion.

Let the screen be in the plane $x = 0$ and let the edges of the slit be at $y = \pm a$. Let the plane wave $u_i = \exp[ik(x \cos \alpha - y \sin \alpha)]$ be incident from the left. We seek a field u which satisfies the reduced wave equation, has a vanishing normal derivative on the screen, is sufficiently regular at the edges and consists of u_i plus outgoing waves. To find u approximately, we assume that it has the form

$$u = u_i + w_1 + w_2 + c_1 v_1 + c_2 v_2 \tag{1}$$

The field w_1 is the field scattered by the upper half-plane when it is hit by u_i in the absence of the lower half-plane. Thus $u_i + w_1$ is the solution of the half-plane problem. The field v_1 is the field scattered by the upper half-plane when it is hit by the field from a unit (line) dipole located at the edge of the lower half-plane, (0, - a), in the absence of that half-plane. The dipole moment is oriented normal to the screen, i.e. parallel to the x-axis. The constant c_1 is to be determined. The corresponding quantities with subscript "2" refer to the lower half-plane.

To determine c_1 and c_2 we shall impose the boundary condition $u_x = 0$ on each half-plane. From the definitions of w_1 and w_2, it follows that $(u_i + w_1)_x = 0$ on the upper half-plane while $(u_i + w_2)_x = 0$ on the lower half-plane. Thus the boundary condition becomes

$$w_{2x} + c_1 v_{1x} + c_2 v_{2x} = 0 \qquad x = 0,\ y > a \tag{2}$$

$$w_{1x} + c_1 v_{1x} + c_2 v_{2x} = 0 \qquad x = 0,\ y < -a \tag{3}$$

Equations (2) and (3) are two linear equations for c_1 and c_2. They have the solution

$$c_1 = (w_{1x}^- v_{2x}^- - w_{2x}^+ v_{2x}^-)(v_{1x}^+ v_{2x}^- - v_{1x}^- v_{2x}^+)^{-1} \tag{4}$$

$$c_2 = (w_{1x}^- v_{1x}^+ - w_{2x}^+ v_{1x}^-)(v_{1x}^+ v_{2x}^- - v_{1x}^- v_{2x}^+)^{-1} \tag{5}$$

In these equations a superscript "+" means that $y > a$ and a superscript "-" that $y < -a$.

These expressions for c_1 and c_2 depend upon the two values of y and therefore they are not constant. But if we use the asymptotic forms of the functions on the right sides of (4) and (5) then these expressions are constant. Therefore this is what we do. The asymptotic forms to which we refer are valid at points which are many wavelengths from the edge of either screen.

Consequently our results will be applicable to slits which are wide compared to a wavelength. Analytically this means that ka must be large. But just as in the case of the soft screen, the results are very accurate even when a/λ is as small as unity.

If ρ is the distance from a fixed point to the edge of one of our screens, then method I requires large $k\rho$, in addition to large ka. Therefore methods I and II should agree when $k\rho$ and ka are both large. They do agree in fact. Thus when the constants determined by (4) and (5), in the manner described above, are inserted into (1), then the asymptotic expansion (for large k) of the resulting formulas lead to the usual geometrical optics field plus a diffracted field. That diffracted field agrees with u_d as given by the sum of (10) and (18) of part I.

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